High-precision Trajectory Tracking Control for Uncertain Nonminimum Phase Hypersonic Vehicles

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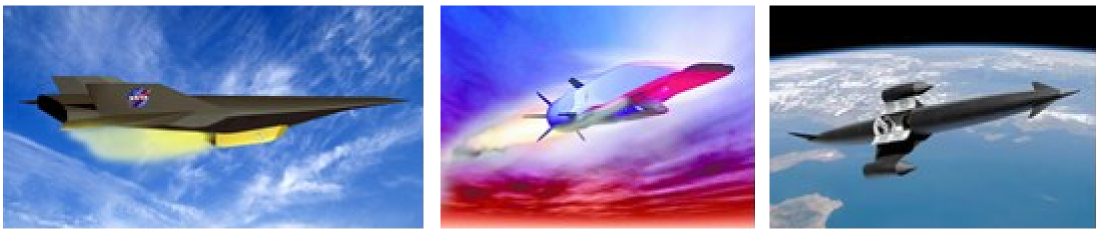
***Abstract*—Hypersonic vehicle (HSV) is one of the cutting-edge technologies in the research area of aerospace and is also an intensively focused strategic direction for global military powers. Precise prompt global strike as the ultimate skill of HSV, the key relies on high-precision trajectory tracking control. Due to the lack of experimental data, the accurate model of an HSV is very difficult to obtain. Besides, the HSV model exhibits nonminimum phase property. These features make it extremely difficult to achieve high-precision trajectory tracking control for an HSV, which has become a bottleneck for the development of the HSV control system. To this end, this paper focuses on high-precision trajectory tracking control for a nonminimum phase HSV, with special emphasis on the existence of model uncertainties. The proposed control framework consists of three parts. Firstly, design of a minimum-phase new output based on virtual constraints and robust optimization. Secondly, calculation of the ideal internal dynamics based on experience replay and optimal bounded inversion. Thirdly, robust nonlinear controller design based on receding horizon. By the end of this research, a control framework is built for high-precision trajectory tracking for nonminimum phase HSVs, which can provide scientific theory and support for the sustainable development of the HSV control technology.**

Keywords—Hypersonic vehicles, Nonminimum phase, Tracking control

I. INTRODUCTION

Hypersonic vehicle refers to the vehicle with flight Mach number greater than 5. They are usually equipped with airbreathing engine or combined engine as the main power, which can fly in the atmosphere and trans-atmosphere for a long range. Their application forms include hypersonic cruise missiles, hypersonic aircraft, space aircraft, and other aircrafts. The typical representatives of hypersonic aircrafts are the X-43A aircraft, the X-51A aircraft from the United States and the Skylon space aircraft from the United Kingdom, as shown in Fig. 1. Due to the extreme-speed advantage, hypersonic vehicles as a weapon system is expected to achieve the goal of "one-hour global rapid strike", and existing missile defense systems are mostly ineffective to intercept it. Therefore, hypersonic vehicles have great strategic importance and military value, and have become the strategic trend and research hotspot of military powers over the world.

Hypersonic vehicles experience a wide space domain as well as a wide velocity domain during flight, and the flight environment is much more complex than that of traditional aircrafts, which is affected by stronger model uncertainties, fast time-varying characteristics, and unknown external interferences, making the design of control systems face great challenges. Moreover, due to the current insufficient understanding of hypersonic aircrafts and the lack of flight data, it has become extremely difficult to establish accurate models of hypersonic aircrafts. In recent years, the control problem of hypersonic aircrafts has received widespread attention. Stengel et al. from Princeton University used the nonlinear dynamic inversion method to design a nonlinear controller for the hypersonic vehicle [1]. Serrani et al. from Ohio State University considered the model of air-breathing hypersonic vehicles with canards and designed the controller by using dynamic inversion method combined with adaptive scheme [2]. Mirmirani et al. from University of Southern California designed an adaptive sliding mode controller for the longitudinal model of hypersonic vehicles by combining feedback linearization and adaptive sliding mode control [3]. Lisa et al. from Ohio State University proposed a new nonlinear sequence loop closure method to design a state feedback controller for hypersonic vehicles [4]. Andy Kurdila et al. from Virginia Tech applied the L1 adaptive control for hypersonic vehicle model, and this method can well compensate for the parameter uncertainties and unmodeled dynamics [5]. In addition, the research on hypersonic vehicles in China has also made great progress in recent years. For example, Xubin et al. from Northwestern Polytechnic University introduced the intelligent adaptive control method into the control system design, making it capable of handling uncertainty, fault tolerance and self-learning [6]. Bu Xiangwei et al. from Air Force Engineering University of PLA proposed a preset performance adaptive backoff control method to ensure that the full state of the aircraft meets the preset transient and steady state performance, which can ensure the transient performance of the aircraft under uncertain conditions [7]. Mu Chaoxu et al. from Tianjin University applied the adaptive dynamic programming method to the hypersonic vehicle and realized the control based on data-driven methods[8]. Zong Qun et al. from Tianjin University



(a) X-43A aircraft (b) X-51A aircraft (c) Skylon Space Shuttle

Fig. 1 Typical hypersonic vehicle

studied the elastic problem of hypersonic vehicle and proposed the design method of elastic observer, which provided a feasible technical approach for the stable control of hypersonic vehicles [9][10].

Although a variety of control methods have been proposed, the United States still experienced many failures in the X-51A and HTV-2 hypersonic flight tests in recent years, which shows that the control technology of hypersonic vehicles is far from mature. A major hazard in hypersonic vehicle control is its non-minimum phase property. Studies have shown that the hypersonic vehicle exhibits non-minimum phase characteristics in the longitudinal model due to the influence of lift-elevator coupling [11]. When the traditional nonlinear control method is directly applied to the hypersonic vehicle, the unstable zero dynamics will be retained in the closed-loop system and the system stability will be destroyed. Therefore, the non-minimum phase characteristic of hypersonic vehicle prevents the application of traditional nonlinear control methods. If the problem is not handled properly, the flight may fail. Recently, much attention has been paid to the control of non-minimum phase hypersonic vehicles. In 2007, Parker et al. from the United States Air Force Laboratory used the approximate feedback linearization method to design a controller [12]. By ignoring the lift-elevator coupling, the controller was obtained by the feedback linearization of the approximate model. This method was only applicable to the case of small lift-elevator coupling. In 2008, Sigthorsson et al. from Ohio State University in the United States designed a robust output feedback controller [13] based on the internal model principle. By solving the regulator equation, the reference trajectories of all state variables and feedforward control variables were obtained, and the trajectory tracking was realized by adding feedforward and linear feedback of state error. In 2012, Fiorentini et al. from Ohio State University designed a controller based on the output redefinition method [11]. Under the B-I standard form of the model, a stable manifold was designed based on the nonlinear small gain theorem to make the zero dynamics stable, and then a nonlinear controller was designed based on the new output obtained from the manifold. In 2013, Hu Xiao-xiang et al. from Xi'an Institute of High Technology designed the optimal feedback controller by using LQR method [14], and used the stable inversion method to obtain the feedforward control input required for accurate tracking, which can realize better trajectory tracking. In 2015, they further designed a fuzzy controller [15] to improve the robustness of the controller. In 2017, they further considered the unknown input nonlinear problem [16] and input delay problem [17], and designed nonlinear adaptive controllers respectively. In 2018, Ji Yuehui et al. from Tianjin University of Technology proposed an approximate output regulation controller considering the problem of velocity and altitude tracking a sinusoidal maneuvering trajectory [18], where a three-layer neural network was used to approximate the solution of the regulator equation, and finally the approximate tracking control was realized. In 2018, Ohio State University's Mannava et al. proposed a modular adaptive control method [19]. In this method, the internal and external dynamics are considered as interconnected systems, and the saturation function is used to achieve local stabilization of the interconnected system. In 2019, Xu Bin et al. from Northwestern Polytechnical University designed a robust adaptive neural network controller [20]. The controller is designed separately for internal and external dynamics, which can achieve internal state stability while ensuring bounded tracking error. Meanwhile, the author of this paper also proposed an extended loop backstepping controller [21] and an output redefinition - dynamic inversion controller [22] for the non-minimum phase hypersonic vehicle in 2017 and 2018 respectively, which solved the stable control problem of the hypersonic vehicle under the influence of weak non-minimum phase and strong non-minimum phase, respectively. The good robustness of the controller is verified by Monte Carlo simulation.

However, the current controller design of non-minimum phase hypersonic vehicles mainly focused on how to ensure the stability of the system, and only a few literatures have considered the problem of high-precision trajectory tracking. Hypersonic vehicle as a killer weapon, its high-precision trajectory tracking control is the key to achieve global fast and accurate strike. Due to the lack of flight data, the accurate model is difficult to obtain, and the non-minimum phase characteristic of the model makes the high-precision trajectory tracking control of hypersonic vehicles difficult to achieve, which has become a bottleneck restricting the development of hypersonic vehicle control technology. Although high precision tracking control of non-minimum phase systems can be achieved by embedding the ideal internal dynamics in the controller, this is only applicable when the model parameters are precisely known. When the model parameters are uncertain, how to achieve high-precision trajectory tracking control of non-minimum phase hypersonic vehicle is still an unsolved problem. Therefore, this paper proposes the research on high-precision trajectory tracking control of non-minimum phase hypersonic vehicle, focusing on the precise tracking

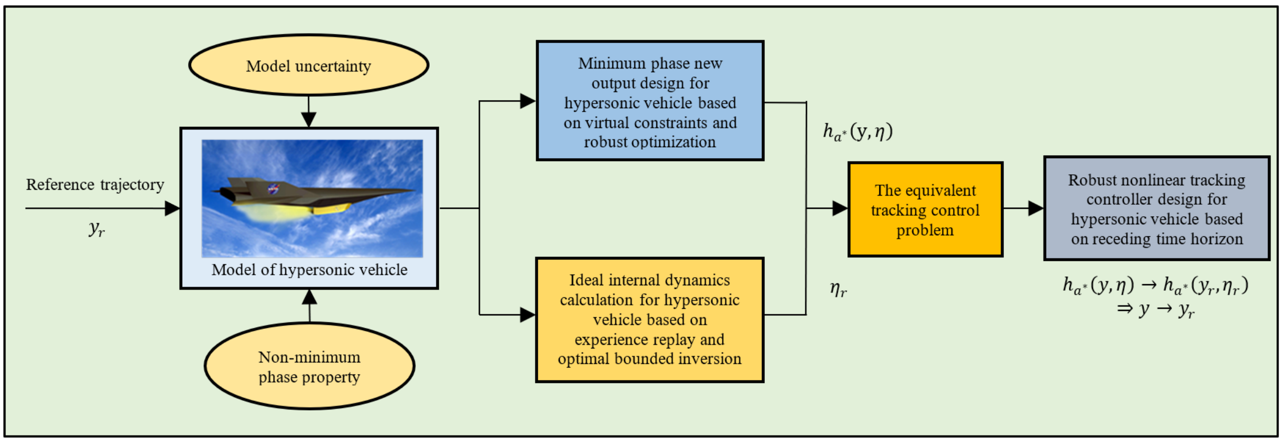


Fig. 2 The overall framework for high-precision trajectory tracking control of uncertain nonminimum phase hypersonic vehicles.

control under model uncertainty. The research of this problem can significantly improve the overall control performance of hypersonic vehicles, and has important scientific significance and wide application prospect for the development of hypersonic vehicles.

II. METHODS

The overall framework for high-precision trajectory tracking control of uncertain nonminimum phase hypersonic vehicles is shown in Fig. 2.

Considering the uncertainties and non-minimum phase characteristics of the hypersonic vehicle model, firstly, a minimum phase new output is designed to stabilize the zero dynamics based on virtual constraints and robust optimization method. Secondly, the ideal internal dynamics of the uncertain non-minimum phase model are solved based on experience replay and optimal bounded inversion method. Thus, the equivalent reference trajectory of the new output is obtained and the original tracking control problem is transformed into an equivalent tracking control problem. Finally, a robust nonlinear tracking controller based on receding time horizon is designed to make the new output to track the equivalent reference trajectory, and thus the original output can also track the given reference trajectory.

A. Design of a Minimum Phase New Output

The design process of a minimum phase new output based on the virtual constraints and robust optimization is shown in Fig. 3.

The first step is the construction of a new parameterized output based on the virtual constraint approach. We establish a parameterized new output expression $h_a(y,\eta)$ with Bernstein polynomial, where y is the original output which represents velocity and altitude, η is the internal state which represents the pitch angle, and a is the coefficient of Bernstein polynomial. Theoretically, aforementioned expression could uniformly approximate any continuous function in a closed interval because of the characteristic of Bernstein polynomial. Therefore, $h_a(y,\eta)$ represents all the possible forms of new output, where the coefficient a represents adjustable parameter for the output, which determines the characteristic of the zero dynamics corresponding to the new output.

The second step is to establish the zero dynamics equation under the virtual constraints. Under the virtual constraint $h_a(y,\eta)=0$, taking the derivative of the new output and setting each-order derivatives to zero, the equations under the virtual constraint is obtained. By solving the above equations, the control input required to maintain the new output at zero and the equation between the external state and the internal state are obtained. Then, the control input is substituted into the internal dynamics equation of the hypersonic vehicle, and the corresponding zero dynamics equation of the new output is obtained. This zero dynamics equation represents the residual dynamics of the system when the new output is zero, which affects the stability of the overall closed-loop system. Since the new output has adjustable parameters, these parameters will be passed to the zero dynamics as well, which determines the convergence performance of the zero dynamics and affects the stability of the closed-loop system.

The third step is to find the best parameters for the new output by using robust optimization. In order to ensure the stability of the zero dynamics corresponding to the new output and make it have good convergence performance, the best value of the parameter a should be determined. Therefore, the problem can be solved by using parameter optimization method. Considering the stability and convergence performance requirements, the minimization for the norm of the zero dynamics is taken as the optimization objective. At the same time, considering the robustness requirements, the model parameters are assumed to have a certain range of uncertainty, which leads to a robust optimization problem. The optimal parameter a^* is obtained by solving this robust optimization problem using a column and constraint generation algorithm. By substituting a^* into the new output expression $h_a(y,\eta)$, we finally get the new output $h_{a^*}(y,\eta)$ which is minimum phase.

B. Solving the Ideal Internal Dynamics

The process of solving the ideal internal dynamics based on experience replay and optimal bounded inversion is shown in Fig. 4.

The first step is online parameter identification based on the experience replay method. In order to make full use of the historical data information of the system, which can speed up the identification process and improve the identification accuracy, the control input and state information during the operation are saved in a data stack. Meanwhile, in order to avoid too much data storage and take into account the timeliness of data, we decide to keep the total number of data points in the data stack unchanged according to the principle of "first in first out". Based on the experience replay method, the historical data and the current data in the data stack are used simultaneously to design the parameter identification law according to the Lyapunov theory to ensure the asymptotic convergence of the parameter estimation value to the true value. Compared with the traditional identification methods that only use the current data, the proposed method makes full use of the historical data information, so its convergence does

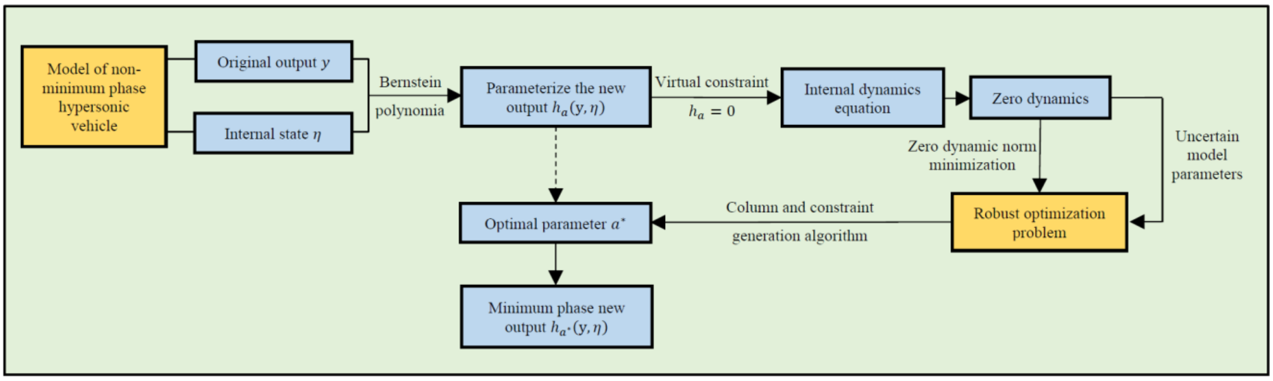


Fig. 3 Minimum phase new output design process based on virtual constraints and robust optimization.

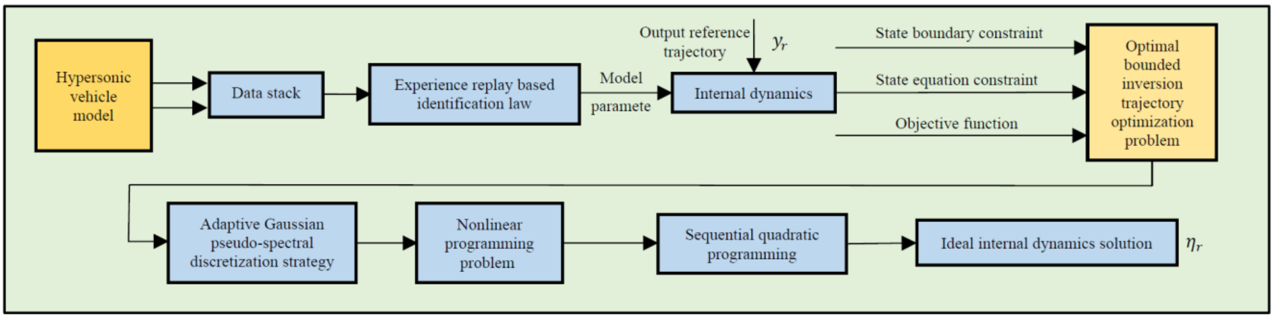


Fig. 4 Ideal internal dynamics solving procedure based on experience replay and optimal bounded inversion.

not depend on the strict continuous excitation condition, and the system state only needs to be stimulated in a short period of time to ensure the convergence.

The second step is to transform the ideal inner dynamics problem into an optimal bounded inversion problem. Considering that the ideal internal dynamics is a bounded solution of the internal state trajectory of the system when the output is constrained to move along the reference trajectory, it can be treated as a trajectory optimization problem. Firstly, the identified model parameters and the output reference trajectory are substituted into the internal dynamics equation as the state equation constraints that the internal state should satisfy. In order to meet the boundedness requirement of the internal state, the maximum and minimum allowable value of the internal state is used as the state boundary constraint. In order to ensure a good match between the trajectory and the initial value of the internal state, the minimum initial value mismatch is selected as the optimization objective. With all the constraints and control objective, the optimal bounded inversion trajectory optimization problem is established.

The third step is to solve the ideal internal dynamics based on the adaptive Gaussian pseudospectral method. In order to solve the optimal bounded inversion trajectory optimization problem quickly, an adaptive Gaussian pseudospectral discretization strategy based on curvature decision and location error is designed. The number and location of the discrete points are adaptively adjusted according to the characteristics of the trajectory. The real-time performance of trajectory solution is improved without affecting the solution accuracy. On the basis of the above discrete strategy, the infinite-dimensional trajectory optimization problem was transformed into a finite-dimensional nonlinear programming problem, and the sequential quadratic programming algorithm can be used to solve it. Therefore, the ideal internal dynamics can be finally obtained with high precision.

C. Robust Nonlinear Tracking Controller Design

The design process of the robust nonlinear tracking controller based on receding time horizon is shown in Fig. 5.

The first step is the design of a robust nonlinear tracking controller. For the nonminimum phase hypersonic vehicle, the tracking controller is designed based on the new output obtained previously. According to the expression of the new output as a function of the original output and the internal state, the given original output reference trajectory and the calculated ideal internal dynamics are substituted into it to obtain the equivalent reference trajectory of the new output, and then the original tracking control problem is transformed into the tracking control problem of the new output to track the equivalent reference trajectory. To achieve this control objective, the design of a robust nonlinear tracking controller is firstly investigated. The robust adaptive backstepping method was used to design the controller, the uncertainty in the system and the external disturbance are taken as the lumped disturbance, which is assumed to be bounded but the upper bound is unknown. Next, an adaptive law is designed based on the Lyapunov theorem to estimate the upper bound of the disturbance, and a robust term with the upper bound estimation is embedded into the controller to compensate the disturbance in real-time.

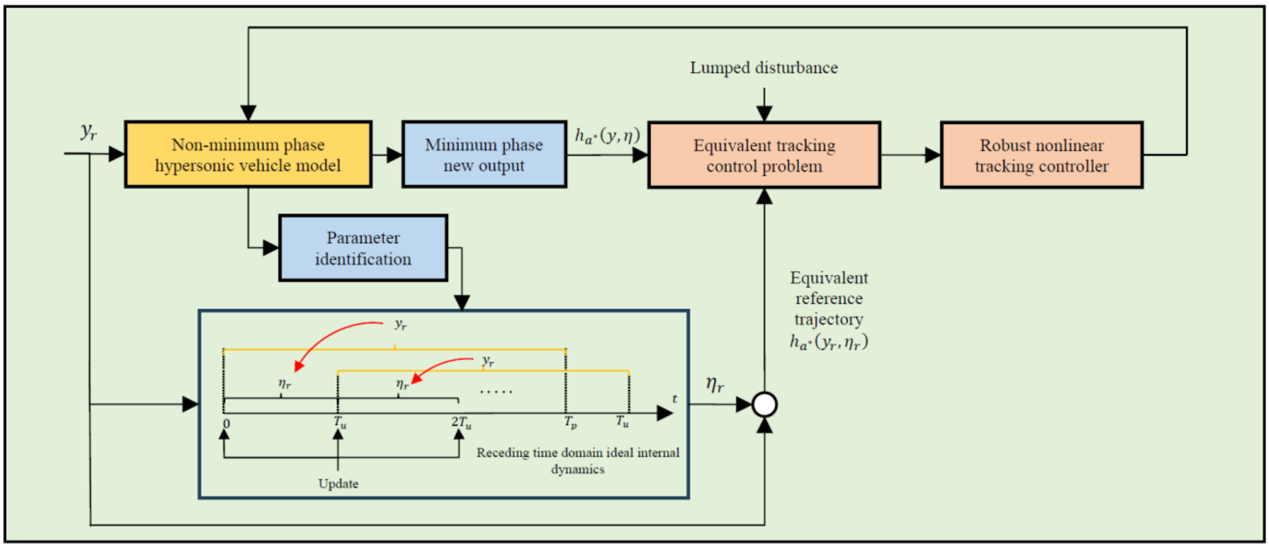


Fig. 5 Robust nonlinear tracking controller design process based on receding time horizon.

The second step is the receding time-domain updating strategy. Considering the influence of uncertain parameters, a time interval of fixed length T_p from the current time to the future is taken, and the ideal internal dynamics within this interval is calculated based on the parameter identification values at the current time. Then, a short part of the obtained ideal internal dynamics in the period T_u ($T_u \ll T_p$) is applied to the controller, and the ideal internal dynamics are updated periodically after each T_u according to the most recent parameter identification values, which ensures high-precision trajectory tracking under model uncertainties.

III. Results

The simulation results are shown in Figs. 6~8. Specifically, Fig. 6 shows the reference trajectories for the two control outputs (velocity and altitude), which are required to track accurately for the hypersonic vehicles. Fig. 7 shows the tracking error with the traditional control method in [11], and Fig. 8 shows the tracking error with our proposed method. It can be seen that the tracking performance with our method is significantly improved. On one hand, the maximum error is greatly reduced, where the velocity error is reduced by about 50% and the altitude error is reduced by about 75%. On the other hand, the tracking error during the middle stage of tracking is very close to zero for our method, while traditional method has a relatively big error in the middle stage. The simulation results verify the effectiveness of the proposed method, which can significantly improve the tracking accuracy for uncertain nonminimum phase hypersonic vehicles.

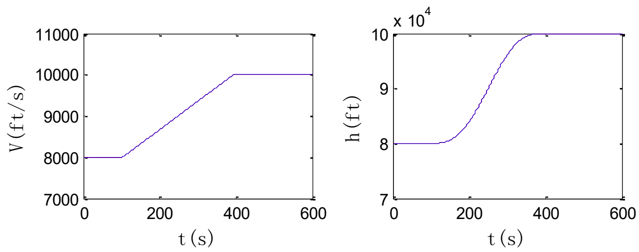


Fig. 6 The output reference trajectories.

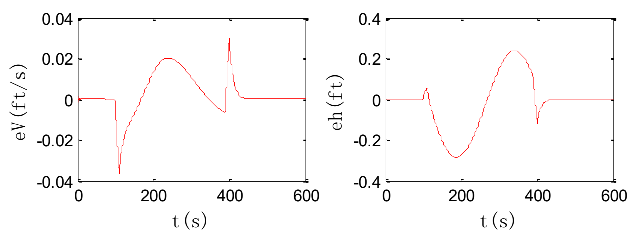


Fig. 7 The tracking error with the traditional control method.

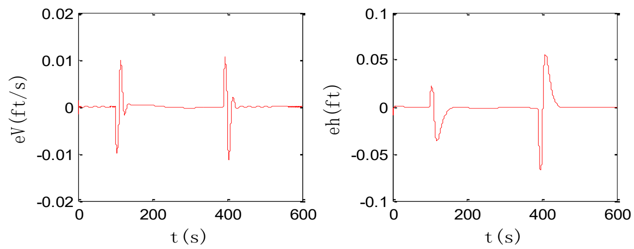


Fig. 8 The tracking error with the proposed method.

IV. Conclusion

In summary, we solve the problem of high-precision trajectory tracking control of nonminimum-phase hypersonic vehicles by proposing a comprehensive control framework. The minimum phase new output design scheme based on virtual constraints and robust optimization effectively improves the zero dynamic convergence performance. The approach of solving the ideal internal dynamic based on experience replay and optimal bounded inversion make full use of the historical data of the system and thus relaxes the dependence on the continuous excitation condition. The design scheme of robust nonlinear tracking controller based on receding time horizon combines the advantages of predictive control and nonlinear control, which ensures the accuracy of trajectory tracking. We verify the effectiveness of the proposed method through the simulation of a trajectory tracking control case. In the future, we still need to consider more situations and do more simulations to further verify the robustness of the proposed method.

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